Gravitational Lensing Signature of Long Cosmic Strings

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The gravitational lensing by long, wiggly cosmic strings is shown to produce a large number of lensed images of a background source. In addition to pairs of images on either side of the string, a number of small images outline the string due to small-scale structure on the string. This image pattern could provide a highly distinctive signature of cosmic strings. Since the optical depth for multiple imaging of distant quasar sources by long strings may be comparable to that by galaxies, these image patterns should be clearly observable in the next generation of redshift surveys such as the Sloan Digital Sky Survey.

An exciting outcome of the interplay between particle physics and cosmology is the realization that topological defects may be present in our universe and may help to resolve some long-standing puzzles such as the origin of structure formation. A particular scenario which has been investigated over the past two decades is one where the relativistic motion of cosmic strings induces large-scale structure formation in the wakes that trail behind them [1]. Such strings would be present even now and, if observed, would be a culminating point for many exciting ideas in particle physics and cosmology. Past research has mostly focussed on the observation of cosmic strings by the anisotropies they produce in the microwave radiation background. In this paper we consider another promising means of detecting strings, namely, by observing the lensing of background galaxies or quasars resulting from strings.

Cosmic strings are lineal gravitating sources with tension along the string equal to the mass per unit length μ. In the case of strings produced at the Grand Unified epoch (cosmic time $t \sim 10^{-35}$ secs), $\mu \sim 10^{22}$ gm/cm. The gravitational effects of such strings are characterized by the dimensionless parameter $8\pi G\mu \sim 10^{-5}$, implying they are strong enough gravitational sources to seed structure formation in the universe. A string network consists of two distinct components: closed loops of string, and long (infinite) strings. Loops are typically of size $\leq \Gamma G\mu t \sim 10^{-4}t$ and live for about one Hubble time. ($\Gamma \sim 100$ is a numerical factor associated with the rate of gravitational radiation.) For infinite strings, numerical simulations of string networks show that there are order 10 long strings within a horizon volume at any epoch. These long strings sweep across the horizon at relativistic speeds and collide and reconnect with other strings. A very important fact, which has emerged after intensive numerical study of the string network evolution, is that these long strings are not straight, but have kinks and wiggles on them [2]. The characterization of this small-scale structure is not fully determined [3] especially since the string simulations do not yet include dissipation of string energy to gravitational radiation.

We should emphasize that the results given in the literature [4] describing the string network are ensemble averaged results. For gravitational lensing by long strings, however, it will turn out that the departures from the ensemble averaged string are all important. For example, even if the average string has fractal dimension close to 1, the sharp discontinuities (kinks) on the string are vital to the gravitational lensing signature of strings. Unfortunately, no suitable characterization of such fluctuations is to be found in the current literature, so we were forced to generate our own ensemble of long strings. The initial string network is generated by laying down random phases on a periodic lattice using the method described by Vachaspati and Vilenkin [5]. Then we use the algorithm devised by Smith and Vilenkin [6] to evolve a network of strings in flat spacetime which allows us to generate a long string sample quickly and efficiently. (A detailed discussion of the numerical simulation will be presented in a forthcoming paper [7].) In using the flat spacetime evolved strings, our hope is that more realistic string ensembles will also yield results qualitatively similar to ours.

Our simulations were run for a time equal to half the light crossing time of the box and so the periodic boundary effects did not become significant. Segments of string within a cube with sides half the length of the box have had sufficient time to relax to a constant, fractal structure, and it is these segments which we use for lensing calculations since longer segments retain the random walk structure of the initial conditions.

After generating a long string, the next step is to find photon trajectories in its gravitational field. The location of the string at time t is given by a vector function $\mathbf{f}(\sigma, t)$ where σ is a parameter along the string. In the absence of the strings, photons would travel from the source to the observer along the trajectory described by their constant initial four velocity γ^μ. When a string is present, the photons are deflected, but, as $8\pi G\mu$ is very small, we need only calculate the bending to first order in this pa-

rameter, that is, in the weak field approximation where we write the metric as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. In our work on lensing by string loops [8], we had shown that for any compact lens, even those moving at relativistic speeds, the photon deflection can be reduced to a single integral along the string on the time slice:

$$t_0 = f_{\|}(\sigma, t_0) \tag{1}$$

where $f_{\|}$ is the component of $\mathbf{f}$ that is parallel to the photon trajectory. To derive the deflection formula in [8], it was assumed that the source and observer reside at distances much larger than the size of the loop, but in the case of long strings, the source and observer are separated by about the horizon size, while the length of string which contributes to the gravitational potential is equally long. We can show, however, that for photons passing near the string, the dominant effect will be produced by only a small fraction of the overall string [7]. So let us consider the deflection arising from a finite segment of an infinite string over which σ ranges from σ_1 to σ_2. In this case the deflection angle is

$$\bar{\alpha}_i = -4G\mu \int_{\sigma_1}^{\sigma_2} d\sigma \left[\frac{F_{\mu\nu}(\sigma,t)\gamma^\mu\gamma^\nu}{1-\dot{f}_{\|}} \frac{f_{\perp i}}{f_\perp^2} \right]_{t=t_0} \tag{2}$$

where, $\mathbf{f}(\sigma, t) = \mathbf{f}_\perp + \mathbf{f}_{\|}$, and, $\mathbf{f}_\perp$ is the component of $\mathbf{f}$ which is perpendicular to the photon trajectory. The index i on α takes on two values to denote components in the plane perpendicular to $\vec{\gamma}$. The tensor $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = \dot{f}_\mu \dot{f}_\nu - f'_\mu f'_\nu - \eta_{\mu\nu} \dot{f}^2. \tag{3}$$

where overdots and primes refer to derivatives with respect to t and σ.

In eq. (2) we have also discarded a boundary term whose effect on the image distortion is suppressed by the ratio of the size of a galaxy to the distance between the galaxy and the string. We assume that the galaxy is at a large distance from the string and are justified in ignoring this contribution to $\bar{\alpha}$.

To calculate the photon deflection in eq. (2), we need to find the string coordinates on the time slice given in eq. (1). For this, we took the unperturbed photon trajectory to be parallel to the z axis and found $f_{\|}$ at all times using our long string evolution code. This allows us to numerically solve eq. (1) and obtain the string trajectory at the particular time slice needed in eq. (2). All our string sections had fractal dimension of about 1.25 within the scale of our 256 by 256 box and above the numerical cut-off scale $\Delta\sigma$ which we set to correspond to a physical scale given by $\Delta\sigma/D_l = 0.1''$ where D_l is the angular diameter distance measured from the observer to the string segment. This choice of $\Delta\sigma$ is small enough for discretization effects to be unimportant in our study of lensing and large enough that the string segment we have constructed stretches across a big enough patch of the sky. (The full 256 by 256 box corresponds to a 25" by 25" square patch of the sky.)

To see how the deflection in eq. (2) causes lensing, consider an axis connecting the observer to the lens, and suppose that there is an image located at a small angular displacement $\mathbf{x}$ from the axis. If we define

$$\alpha = \bar{\alpha} \frac{D_{ls}}{D_s}, \tag{4}$$

where D_{ls} is the angular diameter distance from the lens to the source and D_s is the angular diameter distance from the observer to the source, then the angular location of the source, $\mathbf{y}$, which would produce this image is given by

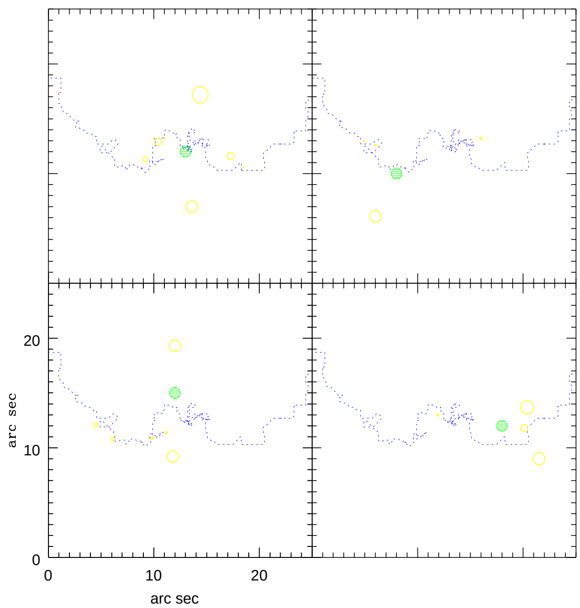
$$\mathbf{y} = \mathbf{x} - \alpha(\mathbf{x}), \tag{5}$$

in the limit of small angles which is always valid here. We have also assumed the thin lens approximation in which the dimensions of the lens are much smaller than D_{ls} or D_s. This is valid as long as we consider short segments of string when compared to the horizon scale.

Before describing our results, it is worthwhile to point out the length scales in the problem. We consider two possible sources for lensing: quasars and high redshift galaxies. Quasars are very luminous, point–like objects with typical redshifts of about $z \sim 2-5$. The typical angular separation of quasars is much larger than that of the typical image separation, so lensed pairs will be distinct. Another set of objects worth observing are galaxies at a redshift of $z \sim 2-3$, with the visible portion of such galaxies having angular diameters of roughly 0.5". The angular separation for the galaxies is about 10" [9]. The angular diameter of the length scale associated with small-scale structure on the string is $\Gamma G\mu t$, which, for a string at a redshift of $z \sim 1$, is very similar to the angular separation of high redshift galaxies corresponding to an angular scale $\sim$ 10" for $\Gamma = 100$. Finally, note also that the mass associated with small scale string structure is $O(10M_{gal})$, so that multiple lensed images could have separations on the order of 10", compared to multiple imaging by galaxies, which generally yields image separations of 1-3".

First we shall consider the lensing of a quasar due to a long cosmic string. In Fig. 1, we show string lensing for four different quasars each located at a redshift of $z = 2$. The projected string configuration is located at $z = 1$, and is shown by the dotted line, where only the contribution from the string shown was used in calculating the image locations. The hatched circle shows the location of the unlensed source, while the open circles show the locations of the various images, where the areas of the open circles are proportional to the magnification of the images relative to the source.

FIG. 1. Several quasar lensing systems. The hatched circle shows the location of the unlensed source and the open circles show the location of the resulting images produced by the string segment (dotted line). The ratio of the areas gives the relative magnification to the source.



In Fig. 2 we show the same projected segment of string as used for the quasar lensing along with the unlensed images of seven circular sources located at redshift $z = 2$. The locations of these sources are chosen randomly in the plane and they have roughly the proper size and density to correspond to high redshift galaxies. In Figure 3 we see the images of these sources as they would appear to a terrestrial observer. (We have set $G\mu = 10^{-6}$ in the simulations.) Note that the 0.1" resolution is consistent with that which can be obtained from the Hubble space telescope, while ground based telescopes achieve resolutions of about 0.5".

It is clear from Figs. 1 & 3 that a sequence of small demagnified images outline the string. One can qualitatively understand this result by considering a chain of point masses separated by distances less than their Einstein radius. Between the masses, the gravitational deflection can be cancelled by the opposite attraction of the two masses. This allows the formation of a small image near the axis connecting the two masses, and for a chain, one might expect to see several small images. In the case of a string, it is the wiggles and kinks which provide the breaks needed to form the small images. The effect is similar to that observed for open string loops [8]; photons passing through a kink are subject to a diminished deflection which can allow an image to be formed.

FIG. 2. The projected string configuration along with the unlensed sources.

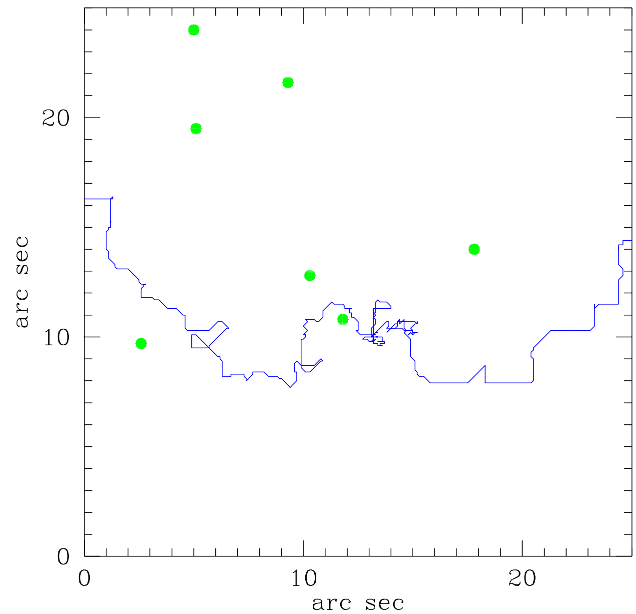
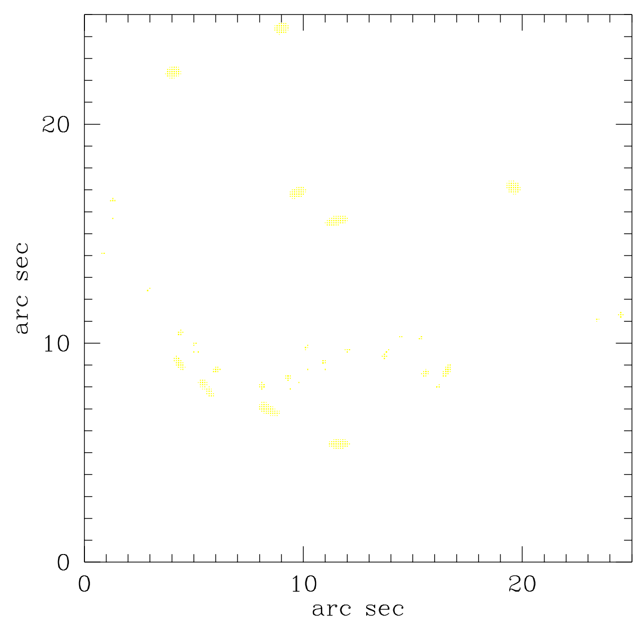


FIG. 3. The resulting images of the sources shown in Figure 2. Only the string points shown in Figure 2 were used in determining the lensing effects.



Thus, the lensing pattern by strings has two distinct signatures: the first is a sequence of baby images that outline the string and the second is a sequence of major images that lie on either side of the string. The first feature depends on the frequency and magnitude of small-scale structure of the string and can be highly distinctive of the stringy nature of the lens. The second feature does

not depend on the small-scale structure of the string and is similar to the previously considered lensing by straight strings due to the conical nature of space-time that they produce.

It should be noted, however, that while the galaxy images in Figure 3 are striking, it is not so clear that actual observed galaxy lensing by strings would be quite so distinctive. In the first place, the galaxy brightnesses are much smaller, so that the small images near the string may not be visible. Secondly, due to the higher space density of galaxies, image separations due to string lensing will be comparable to the separations of galaxies themselves. For both of these reasons, we suggest that a survey involving quasar lensing is probably the most efficient first step in any attempt to probe for lensing due to cosmic strings. As we describe below, galaxies can then provide a useful second probe to confirm the string origin of any quasar lensing.

With that in mind, we consider the probability of observing lensing by long cosmic strings and strategies for observation. As described earlier, it is most useful to first consider surveys of objects which are widely separated on arc second scales, such as quasars. Given that the string density is on the order of ten per horizon volume, the probability of a given high redshift object being lensed by a long string has been estimated to be about $\tau \approx 100G\mu \ln(1+z) \sim 10^{-4}$ [1]. A more refined calculation, based in part on estimates of the long string density arising from numerical simulations [4], and also on the projected angular cross section for string lensing yields an optical depth of $\tau \approx 1. \times 10^{-3}$ (see [7] for further details). To turn this into an actual prediction of lensing frequency in any specific survey requires an analysis of such issues as selection effects, etc. [10]. However, it is useful to compare this optical depth to that due to lensing of high redshift quasars by the known galaxy population. In the ratio of these quantities many such effects should cancel. For a flat universe, the optical depth, assuming the dominant lensing is by elliptical galaxies modeled as isothermal spheres, is $\approx 3 \times 10^{-3}$ [10,11]. If account is taken of possible finite core effects [12], this number could be reduced by a factor of perhaps two. Hence, the ratio of optical depths for string lensing and galaxy lensing in any large sample of quasars is of order unity, suggesting on average as many string induced multiply imaged quasars as galaxy induced ones.

Existing surveys have unearthed on the order of a dozen multiply imaged quasars, so one might expect that there should be several string candidates in this sample. However, because the long string density in our horizon volume is small, a significant fraction of the sky would have to be surveyed before a definitive constraint could be derived (i.e. the string sample is non-Gaussian). A good search strategy would be to observe a large number of quasars in a wide angular field, and the Sloan Digital Sky Survey is precisely such an observation. Approximately 10^5 quasars will be observed over 1/4 of the sky with typical redshifts of about $z \sim 2$. One can expect that the SDSS will observe at least on the order of several hundred lensed quasars, with roughly equal numbers due to galaxies and long strings. If the strings have significant small scale structure, or induce anomalously large angular splittings, one may expect to distinguish the two types of events when the lensing galaxy is not visible, but one cannot necessarily be certain. Here, though, is where galaxy observations can be useful. By using a sensitive telescope like the Hubble Space Telescope, one can observe the high redshift galaxies in the neighborhood of the lensed quasar and look for evidence of string lensing of galaxies. Together with the quasar images, these would likely provide incontrovertible evidence of cosmic strings, or by their absence, rule them out as seeds of structure formation.

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